# WARN!NG S!GNS!

## Recognize three common instructional moves that are generally followed by taking over children's thinking.

By Victoria R. Jacobs, Heather A. Martin, Rebecca C. Ambrose, and Randolph A. Philipp

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Have you ever finished working with a child and realized that *you* solved the problem and are uncertain what the child does or does not understand? Unfortunately, we have! When engaging in a problem-solving conversation with a child, our goal goes beyond helping the child reach a correct answer. We want to learn about the child's mathematical thinking, support that thinking, and extend it as far as possible. This exploration of children's thinking is central to our vision of both productive individual mathematical conversations and overall classroom mathematics instruction (Carpenter et al. 1999), but in practice, we find that simultaneously respecting children's mathematical thinking and accomplishing curricular goals is challenging.

In this article, we use the metaphor of traveling down a road that has as its destination children engaging in rich and meaningful problem solving like that depicted in the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010). This road requires opportunities for children to pursue their own ways of reasoning so that they can construct their own mathematical understandings rather than feeling as if they are mimicking their teachers' thinking. Knowing how to help children engage in these experiences is hard. For example, how can teachers effectively navigate situations in which a child has chosen a time-consuming strategy, seems puzzled, or is going down a path that appears unproductive?

Drawing from a large video study of 129 teachers ranging from prospective teachers to practicing teachers with thirty-three years of experience, we found that even those who are committed to pointing students to the rich, problem-solving road often struggle when trying to support and extend the thinking of individual children. After watching teachers and children engage in one-on-one conversations about 1798 problems, we identified three common teaching moves that generally preceded a teacher's taking over a child's thinking:

1. Interrupting the child's strategy
2. Manipulating the tools
3. Asking a series of closed questions

When teachers took over children's thinking with these moves, it had the effect of transporting children to the answer without engaging them in the reasoning about mathematical ideas that is a major goal of problem solving. We do not believe that any specific teaching move is always productive or always problematic, because, to be effective, a teaching move must be in response to a particular situation. However, because these three teaching moves were almost always followed by the taking over of a child's thinking, we came to view them as *warning signs*, analogous to signs a motorist might see when a potentially dangerous obstacle lies in the road ahead. By identifying these warning signs, we hope that teachers will learn to recognize them so that they can carefully examine these challenging situations before deciding how to proceed.


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## Three warning signs

Consider the following interaction in which Penny, a third grader, is solving this problem:

> The teacher wants to pack 360 books in boxes. If 20 books can fit in each box, how many boxes does she need to pack all the books?

Penny pauses after initially hearing the problem, and the teacher supports her by discussing the problem situation, highlighting what she is trying to find:

***Teacher* [*T*]:** So, she has 360 books and 20 books in each box. So, we're trying to find how many boxes 360 books will fill.
***Penny* [*P*]:** Hmm …
***T:*** So, you have 360 books, right? And what do you want to do with them?
***P:*** Put them in each boxes of 20.
***T:*** Boxes of 20; so you want to separate them into 20, right?
***P:*** Mmm-hmm.
***T:*** Into groups of 20. So, what are you trying to find?

Warning! Even with the best of intentions, some teacher efforts to move students' thinking forward can actually stifle it.

*P:* Trying to find how many go in each—well, you already finded out that, but you need to find how …
*T:* How many boxes, right?
*P:* Right.
*T:* So, you're trying to find out how many groups of 20 there are?
*P:* Mmm-hmm.
*T:* In 360?

After discussing the problem situation, Penny develops an approach, writes 360, and starts incrementing by twenties, writing 20 and 40. At this point, she whispers, "It's gonna take too long," but the teacher encourages Penny to continue by asking about her strategy. "Are you counting by twenties? Is that what you're doing there?"

Penny confirms and resumes her strategy, writing multiples of 20 through 140. Then, from the beginning of her list of numbers, she makes a mark under each one, apparently tallying the number of boxes she has made so far. At the end of her list, she resumes her strategy by writing the next number, 160, and making a mark under it (see **fig. 1**). When Penny pauses briefly before writing the next number, the teacher interrupts Penny's strategy to introduce her own by asking, "Do you know how many times two goes into thirty-six?"

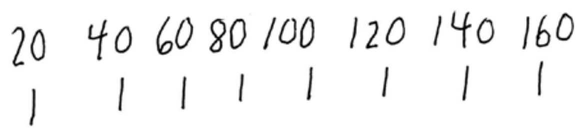
Here we see the first warning sign: *interrupting the child's strategy*. The teacher then picks up a pen and writes the problem 36 ÷ 2 as the standard division algorithm, and we see the second warning sign: *manipulating the tools*. Penny responds, "Twenty," and the teacher invites her to follow the steps to complete the algorithm (e.g., "How many times does two go into three?") but then changes the conversation slightly to consider the original numbers in the problem, writing the division problem 360 ÷ 20 as the standard division algorithm. The teacher completes the first part of the algorithm for this problem herself and then guides Penny through the rest of the steps by asking a series of closed questions, requiring only agreement ("Mmm-hmm") or short answers (e.g., "Eight")—illustrating the third warning sign: *asking a series of closed questions*.

*T:* Do you know how many times 20 goes into 160? [*Penny does not respond.*] Do you know how many times 2 goes into 16?
*P:* Two times sixteen? Times?

**FIGURE 1**

Penny's strategy was to count by twenties.

**(a) She recorded each number, placed a mark under it, and then tallied the marks.**




**(b) When Penny paused, her teacher interrupted and introduced a different approach.**

## MANIPULATING THE TOOLS

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***T:*** Well, if you go, how many 2s are in 16—so, 2, 4, 6, 8, 10, 12, 14, 16 [*writing the numbers while she counts by twos*]. How many is that? [*The teacher points along the list of numbers while she counts aloud.*] 1, 2, 3, 4, 5, 6, 7, 8, right?
***P:*** Mmm-hmm.
***T:*** So, 20 goes into 160, which is just [attaching] a zero. [*The teacher points at the appropriate spot on the paper for Penny to write.*]
***P:*** [*writing*] Eight.
***T:*** Mmm-hmm. Twenty times 8. Yes, 'cause 20 times 8 is 160, so this would be an 8, right?
***P:*** Mmm-hmm.

With the answer of 18 now written, the teacher checks Penny's understanding of what they have just done with another series of closed questions.

***T:*** So, how many boxes do we need? [*When Penny does not respond, the teacher points to the answer of 18.*] What does this represent? Do you know?
***P:*** Eighteen.
***T:*** Mmm-hmm, but do you know like in this problem how we would …
***P:*** Eighty-one? I mean …
***T:*** Do you know what this [18] represents? Like this 20 represents the 20 books that can fit in each box.
***P:*** Mmm-hmm.
***T:*** And 360 represents the total number of books. So, 18 represents …
***P:*** The boxes.
***T:*** How many boxes?
***P:*** Eighteen.
***T:*** There you go. Does that make sense?
***P:*** Mmm-hmm.
***T:*** 'Cause you just have to divide them into the different boxes.

In this example, the teacher began the interaction with moves that supported Penny's thinking (e.g., probing her initial strategy and understanding of the problem) and then helped her reach a correct answer. However, we share this illustration because it also highlights the three moves that should serve as warning signs because they often, and in this case did, lead to taking over the child's thinking: interrupting the child's strategy, manipulating the tools, and asking a series of closed questions.

### 1. Interrupting the child's strategy

When a teacher interrupts a child's strategy to suggest a different direction, the teacher's thinking becomes privileged because the child's thinking—which was "in process"—is halted. This interruption may involve talking over a child who is already speaking, or jumping in when a child is working silently. In both cases, this warning sign generally accompanies the hazard of breaking the child's train of thought—the child may struggle to regain momentum in solving the problem or may lose the thread of his or her idea altogether. Additionally, the teacher may introduce a strategy that does not make sense to the child. In the example above, Penny had a viable strategy and was in the process of executing it when her strategy was interrupted with a different approach proposed by her teacher. Perhaps the teacher thought that Penny's strategy of counting up by twenties would take too long or that she would struggle too much to find each multiple. Or perhaps the teacher had expected (or hoped) that Penny would use the standard division algorithm. In any case, Penny had no opportunity to return to her original strategy and complete it. Furthermore, Penny was making sense of the problem situation with her original strategy, but this sense making disappeared when the teacher introduced the algorithmic strategy.

In our larger study, we observed that some children, like Penny, had viable strategies for solving their problems, whereas other children's strategies and intent were unclear. However, in all cases, their thinking was "in process" in that they were writing, counting aloud, moving fingers while working silently, and so on. The teachers' interruptions sometimes introduced completely new strategies (as in Penny's case) and other times pushed children to engage with their partial strategies in specific ways that changed children's problem-solving approaches and were inconsistent with their reasoning. In each case, teachers risked impeding or aborting children's thinking by inserting and privileging their own ideas while halting the children's in-process thinking.

### 2. Manipulating the tools

Another warning sign teachers should notice is when they visibly take control of the interaction by manipulating the pen, cubes, or other tools. In the example above, Penny had a written recording of her strategy in progress at the top of the page when the teacher's writing of the standard division algorithm shifted Penny's focus to the teacher's strategy. The teacher then retained control of the pen for much of the interaction while she wrote and talked her way through this algorithm. In doing so, she changed the representation of the problem from Penny's written recording of the multiples of twenty and the accompanying tallying of boxes to an approach that was abstract for Penny and not a good match for her thinking—as evidenced in Penny's struggles to make sense of both the calculation and the result.

In our larger study, we observed teachers writing things or moving manipulatives, although sometimes they did so without changing the course of conversations so completely. However, taking over tools was inherently risky because doing so sent children a message about who owned the thinking. Teachers also risked altering problem representations to representations unclear to children—teachers and children may be thinking differently, even when looking at the same manipulatives or written representations (Ball 1992).

### 3. Asking a series of closed questions

This third warning sign highlights a situation that may begin nonhazardously—when the teacher asks a question with a simple and often obvious answer. The danger arises when this question is followed by another and another and another such question. The net effect of a series of closed questions is that the problem gets broken down for the child into tiny steps that require minimal effort and little understanding of the problem situation. Such was the case for Penny after the standard division algorithm was introduced because the teacher asked questions that required little more than Penny's agreement ("Mmm-hmm"). Penny did not have to think about the underlying ideas of division, and the problem-solving endeavor was instead reduced to following directions.

In our larger study, we observed teachers giving directions that were sometimes phrased as questions and other times as steps to follow. In either case, when the answer was finally reached, the children had often forgotten the

**TABLE 1**

Become aware of teaching moves and of potentially taking over students' thinking.

**Warning signs for taking over children's thinking**

| Warning signs | Questions to consider before proceeding | Potential alternative moves |
|---|---|---|
| 1. Interrupting the child's strategy | Do I understand how the child is thinking and will my ideas interfere with that thinking?<br>Will the child be able to make sense of my ideas? | • Slow down: Allow the child to finish before intervening.<br>• Encourage the child to talk about his or her strategy so far.<br>• Ask questions to ensure that the child understands the problem situation and how the strategy relates to that situation.<br>• Ask whether trying another tool or strategy would help. |
| 2. Manipulating the tools | Will the child still be in control of the problem solving?<br>Will my problem representation make sense to the child? | |
| 3. Asking a series of closed questions | Will my questions be about the child's thinking or my thinking?<br>Will the child still have an opportunity to engage with substantive mathematics, or will my questions prevent him or her from doing so? | |


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original goal and were rarely able to relate the solution to the problem situation. We saw this confusion with Penny when she guessed, "Eighty-one?" in response to a question about how many boxes were needed. This apparent stab in the dark was a signal that the teacher's sequence of closed questions did not help Penny make sense of the teacher's algorithmic strategy or relate it to the original problem.

## Heeding the warning signs

The warning signs exemplified in Penny's interaction arose often in our study, sometimes in isolation and sometimes as a set. So, what can teachers do? When possible, we encourage teachers to heed the warning signs by choosing alternative moves that are more likely to preserve children's thinking. The questions in **table 1** are designed to help teachers consider alternative moves. We do not suggest that these alternative moves are foolproof—unfortunately, no moves are. Engaging with children's thinking is a constant negotiation, fraught with trial and error, as teachers work to find ways to elicit and respect children's thinking while nudging that thinking toward reasoning that is more sophisticated. However, in analyzing our data, we were struck with how often the three warning signs were unproductive in achieving this goal, thus prompting us to consider alternative moves.

For example, how might the interaction have been different if Penny had not been interrupted and had been able to complete her initial strategy? The teacher could have probed Penny's completed strategy, validating and eliciting her ways of thinking about the problem. If the teacher still wondered about efficiency, she might have asked if Penny could think of another way of solving the problem, perhaps in a way that was more efficient. This approach would have built on Penny's ways of thinking about the problem while still preserving the goal of efficiency. Alternatively, if the teacher did choose to suggest the division algorithm, she could have left Penny in control of the pen and posed some open-ended questions to explore Penny's understanding of the algorithm and its connection to the problem situation. Another option would have been to ask Penny to consider efficiency while she was still solving the problem with her original strategy. After Penny had completed 160 books (8 boxes) by counting by 20s, the teacher could have asked her to reflect on what she had done so far and if that work could help her proceed more quickly. (This question might prompt Penny to recognize that doubling 160 books [and 8 boxes] would be close to the needed 360 books, but she would also have the option of continuing with her original strategy.) Although there is no perfect move in any situation, these types of alternative moves might have increased the likelihood that the teacher would have supported and extended Penny's thinking without taking over that thinking. (See Jacobs and Ambrose [2008–2009] for more on alternative moves.)

## Are these moves ever productive?

Our data convinced us that the warning signs were generally unproductive moves, but we wondered if these same moves could ever be productive. After all, teaching moves need to be considered in context because the same move can be productive in one situation but unproductive in another. We found that the three warning signs were occasionally used productively but, to us, they almost seemed like different moves because, although they looked similar on the surface, they were coupled with the preservation of children's thinking.

For example, teachers sometimes productively interrupted a child going far off track or engaging in an extremely inefficient strategy by discussing with the child how he or she was thinking. This move was not, as we saw with Penny, used to immediately suggest a different direction but instead deepened the child's (and teacher's) understanding of how the child was thinking about the problem. Similarly, teachers sometimes productively manipulated the tools to help organize the workspace by removing "extra" cubes after ensuring that they were considered "extra" by the child (versus, for example, removing cubes to ensure that the correct quantities were represented). This move provided some organizational scaffolding while preserving the child's way of thinking about the problem. Finally, teachers sometimes productively asked a series of closed questions to check on their understanding of a child's strategy. This move kept the focus on the student's thinking by putting the child in position to confirm or deny what he or she had already done, said, or thought. Thus, we are not suggesting that the three warning signs can *never* be used productively. However, our data overwhelmingly showed that these moves typically led to taking over children's thinking and thus should be used with caution.

## Good intentions

All four authors have had the experience of solving a problem for a child without gaining any idea what the child does or does not understand. We always begin these interactions with good intentions, but other pressures (e.g., shortness of time) or goals (e.g., desire to see the child use a more sophisticated strategy) often derail our efforts. Our data also showed that taking over a child's thinking was not linked to any particular tone or interaction style. In other words, in any given situation, any of us can be tempted to take over a child's thinking.

In summary, avoiding the impulse to take over a child's thinking in one-on-one conversations (either inside or outside the classroom) is challenging. We also recognize that the task becomes even more challenging in social situations like small-group work or whole-class discussions. Nonetheless, in all these instructional situations, the same goals exist: eliciting, supporting, and extending children's thinking. Further, the moves identified as warning signs are likely to thwart efforts to achieve these goals because children get transported to the answer without actually engaging in problem solving. In identifying the warning signs, our hope is that teachers will be more likely to pause and consider alternative moves to avoid the dangers of taking over children's thinking. As a first step, we invite readers to go online (see the **More4U** box to the right) to practice recognizing these warning signs in an interaction with a first grader.

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*This research was supported in part by a grant from the National Science Foundation (ESI0455785). The opinions expressed in this article do not necessarily reflect the position, policy, or endorsement of the supporting agency.*




**Victoria R. Jacobs**, **vrjacobs@uncg.edu**, is a mathematics educator at the University of North Carolina at Greensboro. **Heather A. Martin**, **hmartin@ncbb.net**, and **Rebecca C. Ambrose**, **rcambrose@ucdavis.edu**, are mathematics educators at the University of California–Davis. **Randolph A. Philipp**, **rphilipp@mail.sdsu.edu**, is a mathematics educator at San Diego State University in California. They all collaborate with teachers to explore children's mathematical thinking and how that thinking can inform instruction.

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