
A vignette from a primary grade classroom—at a school classified as 100 percent socioeconomically disadvantaged and with about 40 percent English language learners—furnishes a context for examining the NCTM Process Standards (see the sidebar on p. 358) and illuminates the pathway for implementing the new Common Core State Standards for Mathematical Practice at the elementary school level. We specifically focus on how the teacher integrates problem solving, communication, and reasoning and proof.
By Melanie Wenrick, Jean L. Behrend, and Laura C. Mohs
Ms. Mohs’s class
In the middle of September, eighteen second-grade students are working on four mathematics problems (see fig. 1). The true/false and open number sentences are inspired by the work of Carpenter, Franke, and Levi (2003).

After a few minutes, Laura Mohs takes a quick poll of student responses and asks several youngsters to explain their answers. Thirteen students say that $19 + 11 = 10 + 10 + 9 + 1$ is true, three say it is false, and two students are uncertain. Brianna uses a computational strategy to explain how both sides equal thirty, whereas Serena justifies her answer of “true” by identifying the relationships between the numbers on either side of the equation. Their classmates listen closely to the mathematical arguments (see fig. 2), and the two undecided students nod their heads as they become convinced. Serena refers to the true or false question to solve the problem when the class discusses $19 + 11 = 10 + 10 + \_\_\_\_$. For $17 – 10 = 10 – 17$, six students say true, and twelve students say false. Geri thinks it is false, and Daisy thinks it is true, but neither is able to explain why. Maria and Grace specifically address $10 – 17$ as they share why it is false. Maria says, “Seventeen minus ten equals seven. You can’t take away the seventeen, because there is only ten.”

Grace claims, “Seventeen minus ten equals negative seven. It’s not the same, because it is a lower number.”

José begins to explain why he thinks it is true, changes his mind, and decides it is false.
The hour-long mathematics lesson continues with four story problems to solve; the first three each contain three sets of numbers from which to choose an answer (see fig. 3). The class reads the problems together. Mohs makes sure that everyone understands the situations, and then students work independently or in pairs. Many of them choose to draw pictures. Linking cubes, in groups of ten, are also available. Mohs monitors students as they work, intervening only if someone has trouble getting started, needs assistance recording a strategy, or finishes early. She listens to students’ solutions, asks questions about how they solved their problems, and makes decisions about who will share with the class to generate a productive mathematics discussion. After twenty minutes, students put away the materials and get ready to discuss their solutions.

Mohs selects Alex to share his strategy for the first problem (see fig. 3, question 1). Using the overhead projector, he explains how he drew thirty-seven lines and crossed off twenty-seven of them (see fig. 4a). The other students agree that this strategy works. Andrea draws three large rectangles (tens) and seven smaller rectangles (units) to represent thirty-seven (see fig. 4b). She crosses off 2 tens and 7 units to take away twenty-seven. Mohs asks how the two students’ strategies are the same and how they are different. Their classmates notice that both Alex and Andrea used the same numbers, drew pictures to solve the problem, and got the same answer. Serena remarks that Andrea’s strategy is faster than Alex’s because she did not count all the candies one by one.

The discussion continues as other students explain their strategies for other numbers. Some students use pictures or cubes to model the problems; others use counting. Felix and Grace...
decompose the numbers to make 127 – 79 easier to compute (see fig. 5). Felix represents the 127 as 12 tens and 7 units and then crosses off 7 tens and 7 units. He still needs to take away 2 units, so he crosses off a 10 and makes it into 8 and 2. After crossing off the 2, he counts what is left.

Grace starts with the 120 from the 127 and then takes away the 70 in chunks, first 20 to get to 100 and then the remaining 50, leaving 50. She still must deal with the units. She understands that 7 – 9 is less than zero and that she needs to take that 2 away from the 50, giving her an answer of 48. Lunch interrupts the conversation.

**Problem solving**

Mohs’s second-grade classroom is an engaging learning environment that exemplifies both the NCTM (2000) Process Standards and the Common Core State Standards for Mathematics (CCSSI 2010). Mohs believes in children’s ability to solve problems; she has created a classroom atmosphere that allows children to explore mathematics through problem solving. Mohs writes accessible yet challenging problems that incorporate addition, subtraction, multiplication, and division. By mixing the operations required to solve the problems, Mohs ensures that her students make sense of each problem situation. As the vignette shows, students receive the time and materials to solve problems in their own way. Students feel safe expressing their ideas because their thinking is valued; mistakes are expected and not feared. Understanding that confusion is a natural part of learning motivates them to search for solutions. Because they believe they can solve a problem, they persevere until a solution is found, reflecting one of the Standards for Mathematical Practice, “Make sense of problems and persevere in solving them” (CCSSI 2010, p. 6). Mohs’s class frequently discusses the importance of understanding why something works or whether the solution makes sense. Getting the “right” answer is not enough. “How do you know?” becomes the norm for everyone.

A critical aspect of teaching through problem solving is to choose problems purposefully. To make instructional decisions, Mohs uses research on problem types (Carpenter et al. 1999), her knowledge of her students, the state-adopted content standards, and mathematics principles. She selects problems and numbers on the basis of potential strategies and the purpose of the lesson. She provides several number choices in story problems so students have multiple entry points that accommodate varying levels of understanding. The class talks about how to choose numbers that are challenging but not overwhelming. Students learn how to select different sets of numbers if they need easier or harder numbers or for when they finish early.

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**FIGURE 5**

Felix and Grace both decompose 127 – 79 to make it easier to solve.

**(a) Felix’s strategy (127, 79)**

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**(b) Grace’s strategy (127, 79)**

120 – 20 = 100;
100 – 50 = 50;
7 – 9 = –2;
50 – 2 = 48

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**Process Standards excerpts**

To give context to how this second-grade teacher is implementing the Common Core State Standards for Mathematical Practice, she refers to excerpts from the NCTM (2000) Process Standards.

**Problem solving**

Problem solving is natural to young children because the world is new to them, and they exhibit curiosity, intelligence, and flexibility as they face new situations. The challenge at this level is to build on children’s innate problem-solving inclinations and to persevere and encourage a disposition that values problem solving. (NCTM 2000, p. 116)

**Communication**

An important step in communicating mathematical thinking to others is organizing and clarifying one’s ideas…. Students in prekindergarten through grade 2 should be encouraged to listen attentively to each other, to question others’ strategies and results, and to ask for clarification so that their mathematical learning advances. (NCTM 2000, p. 129)

**Reasoning and proof**

The ability to reason systematically and carefully develops when students are encouraged to make conjectures, are given time to search for evidence to prove or disprove them, and are expected to explain and justify their ideas. (NCTM 2000, p. 122)
Mohs designed the subtraction problem (see fig. 4) to build on decomposing strategies that her students had used with addition problems. She anticipated that some students would pull apart the 107 and 57 and use the fact $100 - 50 = 50$ to solve the problem. She hoped that students working with $37 - 27$ would recognize the difference in the tens and use this knowledge to solve the problem. With the numbers $127 - 79$, she was curious how students who had used decomposition in the past would address the difference in both the tens and units places. During the class discussion, Mohs planned to have students share different decomposition strategies and examine the efficiency of these various approaches. During the actual lesson, Mohs listened closely to students’ strategies, paying particular attention to how her students decomposed numbers. By observing their strategies, she learned that (a) Andrea visualized 37 as 3 groups of ten and 7 units; (b) Alex understood the problem situation but was not thinking of the numbers as groups of tens and units; (c) Felix had a strong understanding of tens and units; and (d) Grace used number relations to simplify her computation. Mohs could see which students were “using different properties of operations and objects” (CCSS 2010, p. 6) by how they chose to break apart the numbers.

Mohs’s mathematics class exemplifies how a problem-solving atmosphere allows all students at different levels to develop their understanding of mathematics by engaging in relatable problem contexts while also deepening their understanding of mathematics concepts. Mohs is constantly making decisions about the problems and how to structure sharing them in ways that help her students learn essential mathematics content. Ultimately, Mohs wants her students to “apply the mathematics they know to solve problems arising in everyday life,… interpret their mathematical results in the context of the situation, and reflect on whether the results make sense” (CCSSI 2010, p. 7).

**Communication**

Communication is another essential aspect of Mohs’s mathematics class. Mohs observes student strategies as they solve problems; she makes decisions about who will share, determines the sharing sequence, and considers questions to ask to develop students’ understanding of mathematics.

Even though the students are using different numbers to solve problems, Mohs’s class recognizes that the relationships in the problem stay the same and that just the quantity has changed. Mohs usually structures the discussion by starting with the least challenging set of numbers (which are the lowest numbers in this vignette, but the lowest numbers are not always the easiest). Students who choose sets of numbers that are more difficult can easily follow this discussion. Because “younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem” (CCSSI 2010, p. 6), Mohs often first calls on someone who used a pictorial strategy (see fig. 5) to set the stage for later strategies that are more complex. This provides a concrete representation to refer to when discussing other strategies. Using the same situation, starting with the numbers that are the least complex, and beginning with concrete representations are strategies that give all students access to the mathematics embedded in the problem solving.

Initially, children communicate through oral language, aided by tools or a visual representation. Over time, children also learn how to effectively communicate their ideas through written language. Mohs scaffolds by recording student strategies on chart paper when they verbally share solutions. She posts these charts in
Reasoning and proof thrive when students are willing to rethink their own answers and are able to prove that they are right. Students become the authority in determining correct or incorrect answers.

Students are willing to share how they solved problems; however, listening to and examining their classmates’ strategies can be challenging for young children. Mohs “recognize[s] that learning to analyze and reflect on what is said by others is essential in developing an understanding of both content and process” (NCTM 2000, p. 130). A teacher helps children develop these skills by establishing expectations and providing purposes for listening and understanding classmates’ strategies. In the vignette, Mohs asked her students to “identify correspondences between different approaches” (CCSS 2010, p. 6) by comparing Alex’s and Andrea’s solutions for $37 - 27$ (see fig. 4). Students identified similarities, such as drawing pictures and getting the same answer. Other prompts, such as, “Share how you solved the problem with a partner,” “Listen for one new strategy you want to try tomorrow,” or “Which strategy was most like yours?” encourage students to be active participants in discussions.

Sharing strategies makes new discoveries public and can result in major shifts for the whole class, as when one student discovers the efficiency of building numbers in tens and units and as the value of that strategy takes root. One child’s genuine excitement in a discovery generates far more interest in a strategy than a teacher ever could. With communication as a central part of the classroom environment, Mohs’s students positively influence one another’s mathematical learning.

Reasoning and proof

In an environment where students become the authority in determining right or wrong answers, reasoning and proof thrive. Students must be able to prove they are right or be willing to rethink their own answers. “Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments” (CCSSI 2010, p. 7). An initial step toward reasoning is the ability to describe one’s thought processes and then recognize whether the process makes sense. Justification builds on examples, nonexamples, and thinking of mathematical properties. The discussion of whether $17 - 10 = 10 - 17$ is true or false shows a range of justification. Geri and Daisy express their beliefs about the equation, but they are unable to explain their thinking. Maria is able to explain why it does not make sense, comparing the answer of $17 - 10 = 7$ to $10 - 17 \neq 7$. Grace is able to build on Maria’s response by comparing the answers to both problems, $7 \neq -7$. Even as these two students prove it must be false, José does not just accept someone’s answer. In his effort to explain why it is true, he recognizes
“a flaw in [his] argument” (CCSSI 2010, p. 7) and decides it is false.

Mohs’s expectations have created “an environment that respects, nurtures, and encourages students so that they do not give up their belief that the world, including mathematics, is supposed to make sense” (NCTM 2000, p. 122). Her students know that she expects them to explain their solutions to problems; the answer is not enough. She treats both right and wrong answers in similar ways, by asking questions:

- How do you know?
- Are you sure?
- How could you check your answer?
- Is there another way?
- Did your classmates get the same answer?
- How do you know which answer is correct?
- Can there be two correct answers?

Without an outside authority to tell them the right answer, students verify answers as they justify why their answers are correct.

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Posing problems to develop content
Mohs’s mathematics class integrates all the NCTM (2000) Process Standards. We examined three central Standards in depth: Problem Solving, Communication, and Reasoning and Proof. Mohs structures her class by posing problems that develop mathematics content. After solving problems in ways that make sense to them, students communicate their ideas. Going beyond communication, students must justify how they solved the problem and why their answer is correct. The Representation and Connections Standards are also essential components in Mohs’s mathematics class. She allows and encourages students to use multiple representations as they solve problems and share strategies. Mohs purposefully chooses connecting problems and asks students to make connections across problems and strategies.

As states adopt and transition to the Common Core State Standards (CCSSI 2010), teachers must consider how to implement the Standards for Mathematical Practice in their classrooms. Although CCSSM provides information and specific examples about how to teach mathematics, elementary school teachers may have difficulty envisioning how to enact these practices, especially when many examples are geared toward older students and higher-level mathematics. By using the NCTM Process Standards, teachers can visualize how to effectively implement the Standards for Mathematical Practice to create a mathematically rich classroom like Mohs’s, where learning important mathematics is meaningful for all students.

REFERENCES

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